# Interference of Light.

## Wave motion.

Any physical entity which varying both in space and time constitute a wave. common example → stone dropping on water surface.

$$\psi(x,t) = f(x-vt) \quad (+\text{ve } x \text{ direct})$$
$$= f(x+vt) \quad (-\text{ve } x \text{ direction})$$

S, S' w.r.t

diff. wave eqn

$$\frac{\partial^2 \psi}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 \psi}{\partial t^2}$$

$$\psi = c_1 f_1(x-vt) + c_2 f_2(x+vt)$$

## Harmonic wave.

The simplest kind of vibration that a particle of the medium can execute is SHM.

$(x-vt) \to$ equivalent length

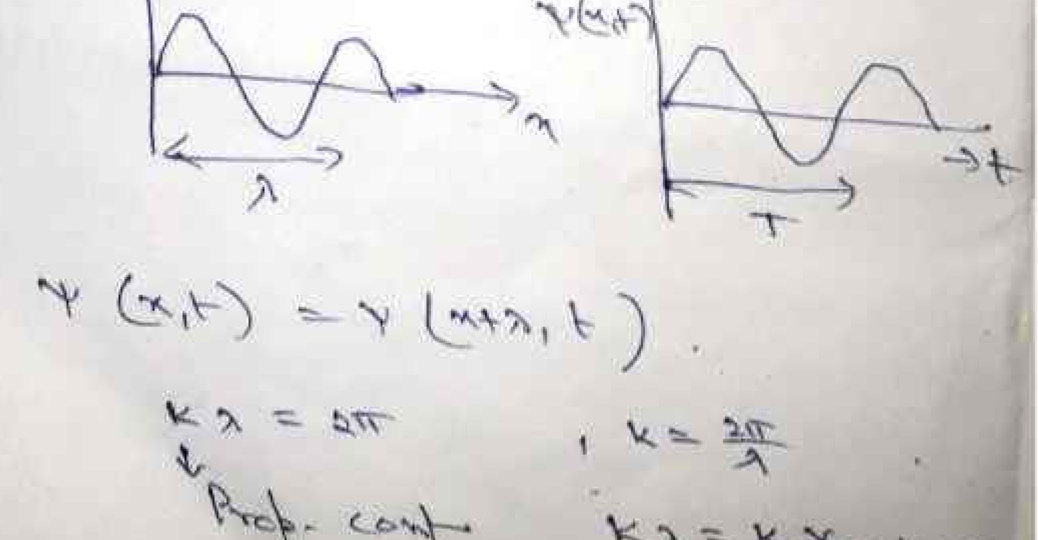
$k(x-vt)$

$$\psi(x,t) = a \sin k(x-vt)$$

$x \to$ dist. equivalent angle $kx$

$$= a \cos k(x-vt).$$

$a \to$ amplitude. , wave is periodic both in space and time.




$$\psi(x,t) = \psi(x+\lambda, t).$$

$k\lambda = 2\pi$ , $k = \frac{2\pi}{\lambda}$

↓

Prop. const

$$k\lambda = k.\frac{v}{n} = kvT = 2\pi$$

$\frac{2\pi}{T} = \omega \rightarrow$ angular frequency

$$\psi(x,t) = a \sin k(x - vt)$$
$$= a \sin(kx - \omega t)$$

$(kx - \omega t) \rightarrow$ phase of the wave.

Let at $x + dx$, $t + dt$, the same phase occur.

$$\phi(x,t) = \phi(x+dx, t+dt).$$

$$kx - \omega t = k(x+dx) - \omega(t+dt)$$

$$kx - \omega t = kx - \omega t + k\,dx - \omega\,dt$$

$$\frac{dx}{dt} = \frac{\omega}{k} = v$$

$v \rightarrow$ velocity of propagation of the ~~wave~~ condition of constant phase. It is called phase velocity.

Wave Front

A wave front is a surface upon which the phase of disturbance is the same at any given instant of time. It is loci of points of constant phase. A wave travels in a direction normal to the wave front.

- point source $\xrightarrow{\text{create}}$ spherical wave front
- Line source $\longrightarrow$ cylindrical wavefront

• Spherical wave front at large distance produces a plane wavefront.

3 dim. wave eqn.

$$\nabla^2\psi = \frac{1}{v^2}\frac{\partial^2\psi}{\partial t^2}$$

Plane wave sol^n

$$\psi = a e^{i(k\cdot r \mp \omega t)}$$

$k\cdot r = \text{const} \rightarrow$ At any given time eqn. of wavefront.

↳ a plane $\perp$ to $\vec{k}$.

Spherical wave.

$$\psi(\vec{r}, t) = \psi(r, \theta, \phi, t).$$

$\psi \rightarrow$ spherically symm. indep of $\theta, \phi$.

$$\therefore \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial\psi}{\partial r}\right) = \frac{1}{v^2}\frac{\partial^2\psi}{\partial t^2}$$

$$\psi = \frac{1}{r} u(r, t).$$

$$\psi(r, t) = \frac{a}{r} e^{i(k\cdot r \mp \omega t)}$$

amplitude decreasing as $\frac{1}{r}$

intensity " " $\frac{1}{r^2}$

Surface of constant phase are $k\cdot r = \text{const}$.

cylindrical wave.

$$\psi(\vec{r}, t) = \psi(r, \theta, z, t)$$

Because of cylindrical symm, $\psi \rightarrow$ indep. of $\theta, z$.

$$\psi = \frac{c}{\sqrt{r}} e^{i(kr - \omega t)} + \frac{c_2}{\sqrt{r}} e^{i(kr + \omega t)}$$

## Huygens' Principle

If we consider a point source of light placed in an isotropic medium, then disturbances propagate in all direction equally, hence if we consider a sphere with the point source as centre, this spherical surface would represent a wavefront at a certain instant.

Huygens principle can be states as — All the particles of wavefront vibrate in phase and these vibrating particles become the sources of secondary disturbance and generate secondary wavelets which spread in the forward direction.

# Interference

When two or more disturbances arrive at a point in space simultaneously the resultant disturbance at that pt, according to superposition principle is given by the vector sum of the disturbances which are assumed to be small.

$$\nabla^2 \psi = \frac{1}{v^2}\frac{\partial^2 \psi}{\partial t^2} \rightarrow \text{represents}$$

propagation of wave.

if $\psi_1, \psi_2 \rightarrow sol^n$

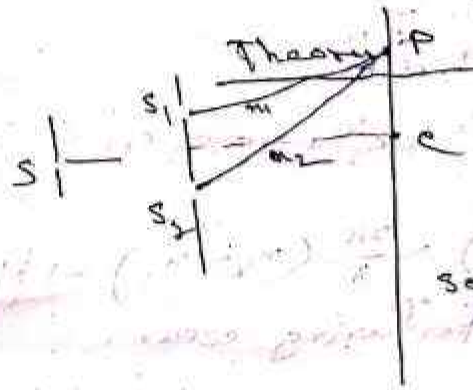
then $\psi_1 + \psi_2 \rightarrow sol^n$ to this eqn in accordance with principle of superposition. Disturbance from each wave at a point is independent of ~~each other~~ presence of the other. When monochromatic waves of light from two sources proceed almost in the same direction and superpose at a point either in same or in opposite phase. Then the intensity of light at that point will be maximum or minimum according as the waves meet the point in same or in opposite phase. This phenomenon is known as interference of light. This phenomenon requires for its explanation that light must have a wave nature.

$S \rightarrow$ illuminate with monochromatic light 7

$S_1, S_2 \rightarrow$ equidistant from $S$.

$y = a_1 e^{i\omega t}$

Theory




Screen.

Two secondary wavelets will start from $S_1, S_2$ and diverge to the screen.

$S_1, S_2 \rightarrow$ act as coherent sources.

Let us represent complex disturbance

$$y = a_1 e^{i\omega t}$$

Starting from $S_1$ to $P$, the phase change occurs $\frac{2\pi}{\lambda} \times S_1P = \frac{2\pi}{\lambda} x_1 = kx_1$

$$y_1 = a_1 e^{i(\omega t - kx_1)}$$

Similarly disturbance at P due to light from $S_2$ —

$$y_2 = a_2 e^{i(\omega t - kx_2)}.$$

$$y = y_1 + y_2 = e^{i\omega t}\left[a_1 e^{-ikx_1} + a_2 e^{-ikx_2}\right]$$

$$= e^{i\omega t}\left[a_1(\cos kx_1 - i\sin kx_1) + a_2(\cos kx_2 - i\sin kx_2)\right]$$

$$= e^{i\omega t}\left[a - ib\right] = e^{i\omega t}\sqrt{a^2+b^2}\, e^{-i\phi}$$

$a = A\cos\theta$
$b = A\sin\theta$
$A e^{-i\theta}$
$\sqrt{a^2+b^2}\, e^{-i\tan^{-1}\frac{b}{a}}$

$$\phi = \tan^{-1}\frac{b}{a}$$

$$a = a_1 \cos kx_1 + a_2 \cos kx_2$$

$$b = a_1 \sin kx_1 + a_2 \sin kx_2$$

$$y = A e^{i(\omega t - \phi)}$$

$$A^2 = a^2 + b^2 = a_1^2 + a_2^2 + 2a_1a_2\cos k(x_2 - x_1)$$

$A \rightarrow$ amplitude of resultant disturbance

Intensity $I \propto A^2$

$$= a_1^2 + a_2^2 + 2a_1a_2 \cos\delta$$

$\delta = k(x_2 - x_1) = \frac{2\pi}{\lambda}(x_2 - x_1) \rightarrow$ phase diff between interfering wave.

$\delta \rightarrow$ depends on path diff.

if $\delta = 2m\pi$

$$x_2 - x_1 = 2m\pi \times \frac{\lambda}{2\pi} = m\lambda$$

$$m = 0, 1, 2, 3, ---$$

Intensity $I$ is max, and proportional to $(a_1 + a_2)^2$

when path diff bet$^n$ two sources is even multiple of $\lambda/2$, the intensity at that point is maximum, we get a bright band at $C$.

If $\delta = (2m+1)\pi$

$$x_2 - x_1 = \frac{\lambda}{2\pi}(2m+1)\pi = \frac{2m+1}{2}\lambda$$

$$m = 0, 1, 2, ---$$

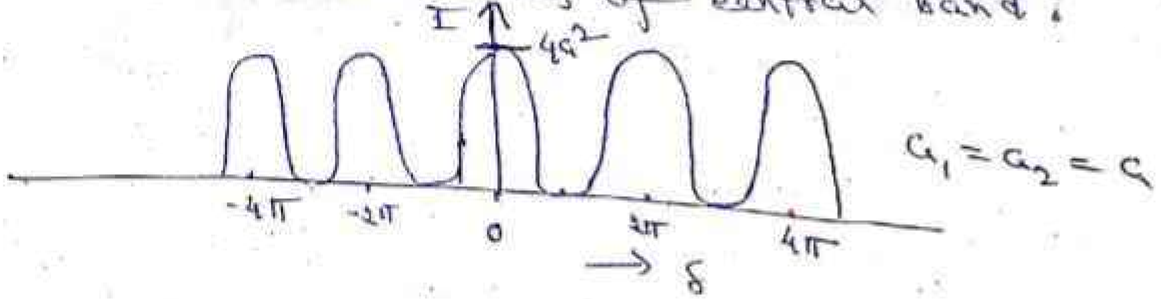
$I$ is minimum and proportional to $(a_1 - a_2)^2$. we get a nearly dark band there. This is known as destructive interference.

when $a_1 = a_2$, minimum intensity zero. we get alternately bright and completely dark band as we go away from the central band.

At C, two wave superpose in same phase. The resultant amplitude is sum of amplitude of individual waves. hence we get a bright band there, which is known as central bright band.

If we take a point $P_1$ on the screen whose path diff from $S_1$ and $S_2$ is $\lambda/2$ then the waves meet at $P_1$ in opposite phase (the crest of the wave from $S_1$ falls on the trough of the wave from $S_2$) and we get a dark band, known as first order dark band.

Again the path diff of another point $P_2$ on the screen from $S_1$, $S_2$ is $2\lambda/2$, the waves meet there in same phase (troughs of both waves fall there), position of next bright band known as the 1st order bright band. Thus as we proceed from C, the path diff changes from odd to even multiple of $\lambda/2$ causing respectively an alternation of minimum and maximum intensity of light on both sides of central band.




Q

i) Is light energy destroyed by interference?

ii) why are light waves from two diff candles not seen to interfere?

Ans: — i) No destruction, but redistribution of energy.

$$I = a_1^2 + a_2^2 + 2a_1a_2\cos\delta$$

$$I_{av} = \frac{\int_0^{2\pi} I\,d\delta}{\int_0^{2\pi} d\delta} = \frac{(a_1^2+a_2^2)(2\pi-0)}{(2\pi-0)} + 0$$

$$= a_1^2 + a_2^2$$

= Sum of intensities of individual waves.

ii) When light waves from two candles meet at a point, the point will be bright or dark according as the waves meet in same or opposite phases. This phase diff. depends on initial diff between the sources and the optical path diff of the point from two sources. The path diff ~~dop~~ remain constant but as two diff candles cannot be coherent, the initial phas ~~continuous~~ changes very rapidly with time. As a result, at every point there is rapid alteration of brightness and darkness and hence we get general illumination.

## Width and Shape of Interference Fringes :

### Fringe Width


$$S_2P^2 = D^2 + \left(x_m + \frac{d}{2}\right)^2$$

$$S_2P = D\left[1 + \left(\frac{x_m + \frac{d}{2}}{D}\right)^2\right]^{\frac{1}{2}}$$

Since $D >> d$, $S_2P = D\left[1 + \frac{1}{2}\left(\frac{x_m + \frac{d}{2}}{D}\right)^2\right]$

Similarly $S_1P = D\left[1 + \frac{1}{2}\left(\frac{x_m - \frac{d}{2}}{D}\right)^2\right]$

For nth ~~b~~ order bright fringe at P

$$S_2P - S_1P = m\lambda$$

$$\frac{1}{2D} 4 x_m \cdot \frac{d}{2} = m\lambda$$

$$\frac{x_m d}{D} = m\lambda$$

$$x_m = \frac{m\lambda D}{d}$$

the distance of $(m+1)$th bright fringe

$$x_{m+1} = \frac{(m+1)\lambda D}{d}$$

distance between two consecutive bright bands

$$\beta = x_{m+1} - x_m$$

$$\boxed{\beta = \frac{\lambda D}{d}}$$

If P be the position of mth order dark band

then $x_m = (2m+1)\frac{\lambda D}{2d}$

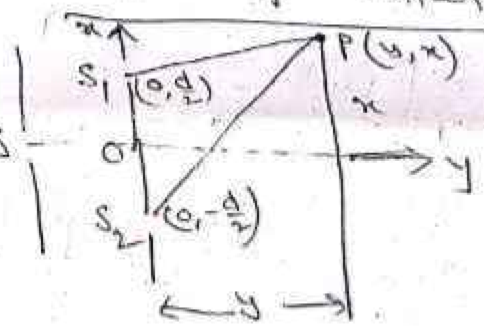
$$x_{m+1} = (2m+2+1)\frac{\lambda D}{2d}$$

$$\beta = x_{m+1} - x_m = \frac{\lambda D}{d}$$

So, it concludes that $\beta$ between two dark or bright bands are equal.

$$\boxed{\beta \to \underline{\text{Fringe Width}} = \frac{\lambda D}{d}}$$

Shape of Interference fringes.




$$S_2P^2 = y^2 + \left(x+\frac{d}{2}\right)^2$$

$$S_1P^2 = y^2 + \left(x-\frac{d}{2}\right)^2$$

$$\Delta = S_2P - S_1P$$

$$\Delta + \left[y^2 + \left(x-\frac{d}{2}\right)^2\right]^{\frac{1}{2}} = \left[y^2 + \left(x+\frac{d}{2}\right)^2\right]^{\frac{1}{2}}$$

$$2\Delta\left[y^2 + \left(x-\frac{d}{2}\right)^2\right]^{\frac{1}{2}} = 2xd - \Delta^2$$

Squaring again and re-arranging

$$\frac{x^2}{\Delta^2/4} - \frac{y^2}{(d^2-\Delta^2)/4} = 1$$

Thus the loci of points of constant path diff. $\Delta$ in xy plane are hyperbola with $S_1, S_2$ as foci on x-axis. The eccentricity of the hyperbola is

$$e = \left[\frac{\Delta^2}{4} + \frac{d^2-\Delta^2}{4}\right]^{\frac{1}{2}} \div \frac{\Delta}{2} = \frac{d}{\Delta}$$

If we use two point source $S_1, S_2$ in 3-dim. space, bright fringes of different order number will form a number of confocal hyperboloids with focus at $S_1, S_2$. if you place a screen parallel to $S_1, S_2$ then short st line fringes (|| to $S_1S_2$) will be obtained.

if screen $\perp$ to $S_1 S_2$, we get concentric bright and dark circles with centre at pt of intersection of $S_1 S_2$ with screen.

## Interference with white light and colour effect:

The distance of mth bright fringe from the central one is given by

$$x_m = m\frac{\lambda D}{d}$$

$$x_m \propto \lambda \Rightarrow$$

$m=0 \Rightarrow x_m = 0$, $\Rightarrow$ bright fringes of all wavelengths will coincide at centre and the central band will be white.

For $m>0$, $x_m \rightarrow$ greater for red light

$\rightarrow$ less for violet light.

fringes will be coloured with red circle outside and violet inside.

When the path difference between interfering waves is large, the condition for constructive interference for one wavelength and the condition for destructive interference for another wavelength may be satified at the same point. In that case the resultant illumination cannot be distinguised from white light. So for observable fringes with white light the path difference should be kept very small.

Condition for observable interference Pattern.

i) The two source of light must be coherent.

ii) The interfering waves must have the same freq. Also, their amplitudes must be equal or very nearly equal.

iii) The original source must be monochromatic or very nearly monochromatic.

iv) The two interfering beams must propagate along the same direction or must intersect at a very small angle.

v) For interference with polarised light the waves must be in the same state of polarisation

Two classes of interference.

classification depends on how we produce coherent sources.

i) Division of wavefront:

e.g. Biprism,

ii) Division of amplitude:

Newton's Ring, Michelson interferometer, Fabry – Perot interferometer